

# Technologies for Energy Production

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Energy cannot be created or destroyed, only converted from one form to another. All life, including humans, relies on this conversion to sustain itself. Humans, however, have taken the conversion of energy further than any other animal on earth. We have figured out how to use energy to build a civilization. Almost every part of modern civilization requires access to energy. However, our current ways of generating useful forms of energy are producing unsustainable amounts of pollution. We have therefore started to develop new technologies to satiate our ever growing hunger. In this paper, I go through the physics underlying different new technologies to produce useful energy. I also touch on advantages and disadvantages with each technology, as well as current costs and hopes for the future.

## I. INTRODUCTION

Ever since we first learned to control fire around one million years ago, energy generation has played a crucial role in our lives [1]. In fact, evolutionary biologists theorize that cooking food with firewood was what allowed our brains to develop in the first place, and there certainly would not have been any industrial revolution without the help of hydrocarbons. Throughout history, burning has been our main source of energy. Burning refers to a chemical process that combines a fuel with oxygen to create energy, water, and carbon dioxide.

For the first one million years of producing energy, we burned plants. Wood in particular provided the majority of the fuel, as it is a dense source of hydrocarbons easily accessible in most parts of the planet. In the last 200 years, we have moved to denser sources of fuel to satiate our growing hunger for energy. Compressed plants in the form of coal, oil, and natural gas now provide the majority of our fuel for burning. Burning has been immensely helpful for our species, but it comes at the price of pollution. In light of this, we are now trying to produce energy in ways that generate less pollution.

Mitigating our pollution of planet Earth is seen by many as the defining challenge of the 21<sup>st</sup> century. Addressing this challenge inevitably involves transforming the way we generate energy. As a result, there are multiple new technologies on the rise. Understanding the underlying physics of these technologies can help us make better decisions about where to target our efforts in the monumental transition we have ahead of us.

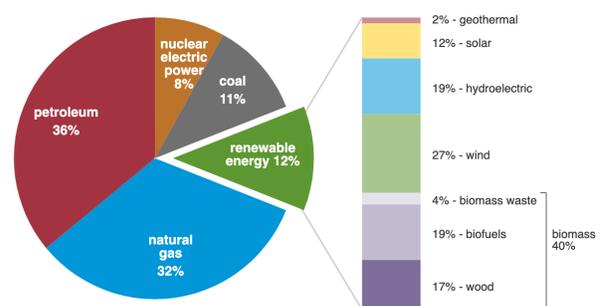
## II. CURRENT ENERGY SOURCES AND COSTS

Right now, burning still dominates energy production in the United States. As is shown in figure 1, renewable energy only make up about 12% and nuclear only 8%. Natural gas and petroleum each provided about 1/3 of the energy consumed in 2021. Coal used to play a much

### U.S. primary energy consumption by energy source, 2021

total = 97.33 quadrillion  
British thermal units (Btu)

total = 12.16 quadrillion Btu



Data source: U.S. Energy Information Administration, *Monthly Energy Review*, Table 1.3 and 10.1, April 2022, preliminary data  
Note: Sum of components may not equal 100% because of independent rounding.

FIG. 1. US primary energy consumption 2021. Figure from U.S Energy Information Administration [2].

larger part in our energy infrastructure, but its share has fallen in recent years mostly due to natural gas becoming cheaper [4].

However, there is reason to believe the distribution is changing. As shown in figure 2, renewable sources have reached the point where they are cost-competitive with fossil fuels and nuclear. To clarify, the numbers in figure 2 show the cost for projects going into service in 2027. Generating power from renewables would be come more expensive when the best wind/solar locations are already taken.

Also note that the cost at which you can provide power is not the sole determinant in investment decisions. It also matters what you can sell the electricity at and how far you have to transport it. In general, because we can choose when to run fossil fuel plants and where to put them, they beat renewables when it comes to the selling price and transport losses. Fossil fuels thus still have an economic advantage in many situations.

Due to the efficiency limits inherent in burning we will discuss later, replacing fossil sources is closer than figure 1 makes it seem. Only a fraction of the energy consumed from petroleum or natural gas is actually used

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Table 1b. Estimated unweighted levelized cost of electricity (LCOE) and levelized cost of storage (LCOS) for new resources entering service in 2027 (2021 dollars per megawatthour)

Plant type	Capacity factor (percent)	Levelized capital cost	Levelized fixed O&M <sup>a</sup>	Levelized variable cost	Levelized transmission cost	Total system LCOE or LCOS	Levelized tax credit <sup>b</sup>	Total LCOE or LCOS including tax credit
<b>Dispatchable technologies</b>								
Ultra-supercritical coal	85%	\$52.11	\$5.71	\$23.67	\$1.12	\$82.61	NA	\$82.61
Combined cycle	87%	\$9.36	\$1.68	\$27.77	\$1.14	\$39.94	NA	\$39.94
Advanced nuclear	90%	\$60.71	\$16.15	\$10.30	\$1.08	\$88.24	-\$6.52	\$81.71
Geothermal	90%	\$22.04	\$15.18	\$1.21	\$1.40	\$39.82	-\$2.20	\$37.62
Biomass	83%	\$40.80	\$18.10	\$30.07	\$1.19	\$90.17	NA	\$90.17
<b>Resource-constrained technologies</b>								
Wind, onshore	41%	\$29.90	\$7.70	\$0.00	\$2.63	\$40.23	NA	\$40.23
Wind, offshore	44%	\$103.77	\$30.17	\$0.00	\$2.57	\$136.51	-\$31.13	\$105.38
Solar, standalone <sup>c</sup>	29%	\$26.60	\$6.38	\$0.00	\$3.52	\$36.49	-\$2.66	\$33.83
Solar, hybrid <sup>d,e</sup>	28%	\$34.98	\$13.92	\$0.00	\$3.63	\$52.53	-\$3.50	\$49.03
Hydroelectric <sup>f</sup>	54%	\$46.58	\$11.48	\$4.13	\$2.08	\$64.27	NA	\$64.27
<b>Capacity resource technologies</b>								
Combustion turbine	10%	\$53.78	\$8.37	\$45.83	\$9.89	\$117.86	NA	\$117.86
Battery storage	10%	\$64.03	\$29.64	\$24.83	\$10.05	\$128.55	NA	\$128.55

Source: U.S. Energy Information Administration, *Annual Energy Outlook 2022*

<sup>a</sup>O&M = operations and maintenance

<sup>b</sup>The tax credit component is based on targeted federal tax credits such as the Production Tax Credit (PTC) or Investment Tax Credit (ITC) available for some technologies. It reflects tax credits available only for plants entering service in 2027 and the substantial phaseout of both the PTC and ITC as scheduled under current law. Technologies not eligible for PTC or ITC are indicated as NA, or not available. The results are based on a regional model, and state or local incentives are not included in LCOE and LCOS calculations. See text box on page 2 for details on how the tax credits are represented in the model.

<sup>c</sup>Technology is assumed to be photovoltaic (PV) with single-axis tracking. The solar hybrid system is a single-axis PV system coupled with a four-hour battery storage system. Costs are expressed in terms of net AC (alternating current) power available to the grid for the installed capacity.

<sup>d</sup>As modeled, we assume that hydroelectric and hybrid solar PV generating assets have seasonal and diurnal storage, respectively, so that they can be dispatched within a season or a day, but overall operation is limited by resource availability by site and season for hydroelectric and by daytime for hybrid solar PV.

FIG. 2. Levelized cost of electricity (LCoE) for projects going into service 2027. Data from the EIA [3].

in useful ways [4]. More than half of it is lost to heat. Renewables can be much more efficient. For petroleum, which is almost exclusively used in transportation, only about 1/3 goes to moving the wheels of vehicles [4]. This means a doubling of our current renewable capacity, in combination with electrifying vehicles, would be enough to completely replace the energy provided by petroleum.

### III. FARADAY'S LAW

Every current method of generating electricity, except photovoltaic solar, ultimately relies on Faraday's law for conversion into electricity. Faraday's law is an expression relating the electric field  $E$  to the magnetic field  $B$ . It's one of Maxwell's four equations and can be expressed both in a differential and integral form:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \iff \oint \vec{E} \cdot d\vec{r} = -\frac{\partial \Phi}{\partial t} \quad (1)$$

where  $\Phi$  is the total magnetic flux through the area enclosed by the line integral. Faraday's law is fundamental to our modern world. By doing mechanical work to change the magnetic flux through a loop of wire, a non-zero electric field inside the wire is created. An electric field means charge will flow and work can be done. The potential to do work  $V$  between two points  $p_1, p_2$  is defined actually defined as the line integral of the electric field between the points:

$$V_{p_1 \rightarrow p_2} = \int_{p_1}^{p_2} \vec{E} \cdot d\vec{r} \quad (2)$$

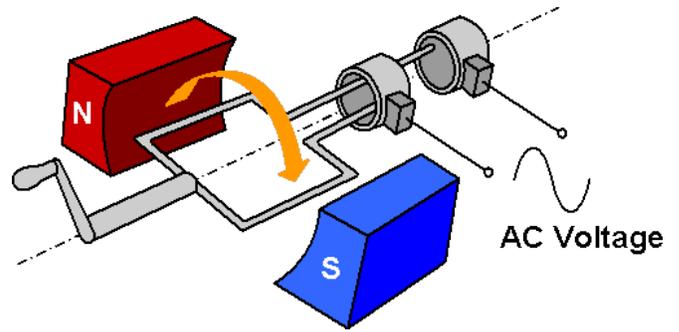


FIG. 3. Picture of AC current generated from spinning a wire in a magnetic field [5]. Works has to be done to keep spinning the wire, as the induced magnetic field will oppose the external one by Lenz's law.

Changing a magnetic field thus creates a potential difference, just like a battery does. Magnetic fields are easy to come by. They can be found emanating from common elements in nature. Many people use them to stick things onto their fridges. The same kind of magnets are used in power plants. The trick is to make the field change with time. All the work we put into making electricity, from burning fuel, splitting atoms, or falling water, goes into changing magnetic fields with time. Without time variation, there is no curl of the electric field and hence no electricity. The magnets can spin around a wire, or the wire can spin around between the magnets. For an example of the latter, see figure 3 [5].

If we take the area of the loop in figure 3 to be  $A$ ,  $\theta$  to be the angle between a vector perpendicular to the wire area and the magnetic field vectors, and  $B$  to be the strength of the magnetic field everywhere between the magnets, then the flux  $\Phi$  through the loop is given by:

$$\Phi = BA \cos \theta \quad (3)$$

where the  $\cos \theta$  term comes from the loop area being at angle  $\theta$  to the magnetic field vectors. We can compute the voltage generated by rotating the wire at angular velocity  $\alpha$  through applying Faraday's law to this flux:

$$-\frac{\partial \Phi}{\partial t} = -BA \cos(\theta) \frac{d\theta}{dt} = -\alpha BA \sin(\theta) \quad (4)$$

Note that this process naturally generates an alternating voltage. As it varies over its range, the potential goes from positive to negative.

### IV. FUEL COMBUSTION

The majority of our energy today comes from burning things. We burn to spin wires in magnetic fields, to move wheels on our vehicles, and to heat our homes. Burning is a terrific source of energy. The only downside is the

pollution it creates. The exact amounts of pollutants vary by fuel, but all fuels release some combination of carbon dioxide, carbon monoxide, sulfur dioxide, and particulate matter. In comparing our current fuel sources, coal releases the most pollutants per unit energy and natural gas the least [6]. Carbon dioxide has risen to fame recently as the cause of global warming, but there are many other ways in which burning pollutes our planet. Sulfur dioxide creates acid rain that damages our forests and buildings. Particulate matter can penetrate deep into our lungs and brain, causing cancer and respiratory diseases.

Classical burning is a quite simple chemical process: hydrocarbons combine with oxygen to create water and carbon dioxide. The energy comes from lowering the electrostatic potential of the electrons involved in the reaction. Negative electrons are on average closer to positive protons in the products than in the reactants. Bringing opposite charges closer together releases energy. The work to separate charges was originally done by plants, plankton and algae using the energy in electromagnetic waves from the sun. Burning can thus be thought of as stored solar power.

Most of the electric potential energy stored in the hydrocarbons is not even used for productive purposes. In fact, it can't be. There is a theoretical limit on the efficiency of generating work from combustion. It's called the Carnot limitation and comes from the cyclic nature of combustion engines. In the expanding phase, it is theoretically possible to reach 100% efficiency, but unavoidable losses occur as the requirement to return to the starting state is imposed. Almost all combustion engines are required to return to the starting state because it would be infeasible and dangerous to have them constantly moving. Jet engines are a notable exception, but they suffer from other efficiency limits that are beyond the scope of this paper.

### The Carnot Limit

To derive the Carnot limit, we will analyze a simple heat engine: a single piston pushed by a heated gas. This type of engine is also referred to as a Carnot Engine. See figure 4. The pressure of the gas will be denoted by  $P$ , the temperature by  $T$ , and the volume by  $V$ . In the expanding phase of the Carnot Engine, the line marked as *I* in figure 5, a heat source keeps the gas temperature constant as it expands. The work done by the gas can be computed using  $\int PdV$ . To evaluate this integral, we define the temperature in this phase as  $T_h$ , apply the ideal gas equation  $PV = nRT$ , and integrate from the volume before expansion  $V_1$  to the volume after expansion  $V_2$ .

$$\int_{V_1}^{V_2} \frac{nRT_h}{V} dV = nRT_h \ln\left(\frac{V_2}{V_1}\right) \quad (5)$$

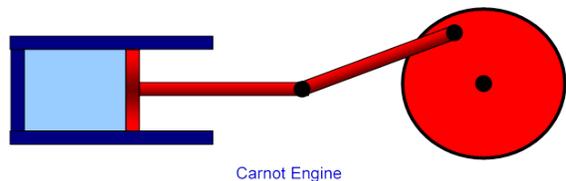


FIG. 4. Schematic of a Carnot engine. The chamber is the blue part on the left and the gas is the light blue inside. A heating or cooling source is placed next to the chamber to enable the gas to do work. The piston is the red part next to the gas, and in this instance it is connect to the red wheel on the left. Note the similarity with the internal combustion engine in figure 7.

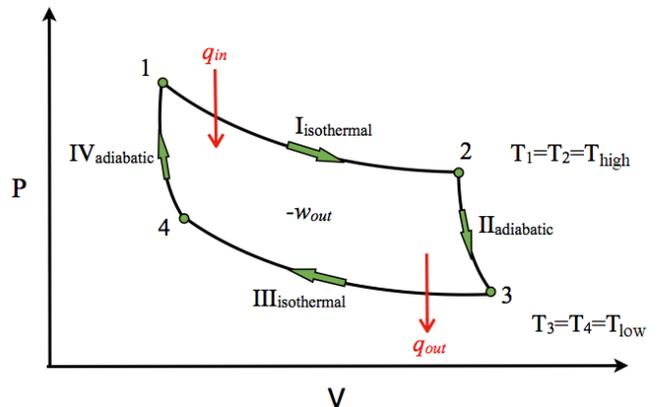


FIG. 5. Pressure-Volume diagram of the Carnot cycle. The parts of the cycle where temperature is held constant are called isothermal, and the parts where no energy is added are called adiabatic.

This is the part of the process that can be 100% efficient in theory. In reality there will always however always be losses due to friction and heat leaks, but we ignore those in this attempt to establish an upper bound on efficiency.

In phases *II* and *IV* in figure 5, no heat is added or removed. This the definition of adiabatic. The positive work in phase *II* is completely canceled by the negative work in phase *IV*. We can understand this by observing that the energy to do work in phase *II* has to come from the internal heat energy in the gas. Since the magnitude of the temperature change is the same in phase *II* and *IV* but the sign is opposite, we can conclude that the work done by the gas as it cools is the same as the work done to the gas to heat it up. Hence the sum of these phases provide no net contribution and we can safely ignore them in our efficiency analysis.

Phase *III* represents compression of the gas. In contrast to phase *I* where the gas did work to move the piston, in phase *III* the piston needs to do work to compress the gas. The work done to compress the gas is the reason for the efficiency limit on Carnot engines. To

be able to return the engine to its initial state, we must provide a reservoir that keeps the temperature constant at  $T_l$  during this phase. Without using the reservoir to cool the gas as it is compressed, the pressure would be higher when the initial volume is reached or vice versa. To find the work in this phase we can use the same integral as previously, but now with  $T_l$  as temperature and from volume  $V_3$  to  $V_4$ .

$$\int_{V_3}^{V_4} \frac{nRT_l}{V} dV = nRT_l \ln\left(\frac{V_4}{V_3}\right) \quad (6)$$

The difference between the work done by the gas and the work done to the gas represents the net work  $W$  the system can output. This is expressed as:

$$W = nR \left( T_h \ln\left(\frac{V_2}{V_1}\right) - T_l \ln\left(\frac{V_4}{V_3}\right) \right) \quad (7)$$

To simplify this expression further, we must establish a relation between the ratios of the volumes. This can be done by analyzing the adiabatic phases *II* and *IV*. In phase *II*, the gas expands and does work. The energy to do work comes from the kinetic energy of the molecules in the gas. At temperature  $T$ , the internal kinetic energy in the gas is proportional to temperature:  $\frac{3}{2}nRT$ . Every incremental bit of work done corresponds to a decrease in temperature. This gives us the relation:

$$\frac{3}{2}nR\Delta T + \frac{nRT}{V}\Delta V = 0 \quad (8)$$

Canceling and taking the limit gives us:

$$\frac{3}{2} \int_{T_h}^{T_l} \frac{1}{T} dT + \int_{V_2}^{V_3} \frac{1}{V} dV = 0 \implies \quad (9)$$

$$\ln\left(\frac{T_l}{T_h}\right)^{3/2} + \ln\left(\frac{V_3}{V_2}\right) = 0 \implies \quad (10)$$

$$\frac{V_3 T_l^{3/2}}{V_2 T_h^{3/2}} = 1 \quad (11)$$

We can do an identical analysis on the other adiabatic phase *IV*. In this phase we go from low temperature to high, and from  $V_4$  to  $V_1$ . Hence  $T_l$  and  $T_h$  switch places,  $V_2 \rightarrow V_4$  and  $V_3 \rightarrow V_1$ . The two equations are thus:

$$\frac{V_3 T_l^{3/2}}{V_2 T_h^{3/2}} = 1 = \frac{V_1 T_h^{3/2}}{V_4 T_l^{3/2}} \implies \quad (12)$$

$$\implies \frac{V_3}{V_2} = \frac{V_4}{V_1} \implies \frac{V_4}{V_3} = \frac{V_1}{V_2} \quad (13)$$

With this relation we can simplify the expression for work in equation 7 to:

$$W = nR \ln\left(\frac{V_1}{V_2}\right) (T_h - T_l) \quad (14)$$

Efficiency  $\eta$  is defined as the work  $W$  done by the system divided by the energy put into the system. The energy put into the system is the same as the work done by the system in the expanding phase since the gas stayed the same temperature during the expansion.

$$\eta = W/Q_{in} = \frac{nR \ln\left(\frac{V_1}{V_2}\right) (T_h - T_l)}{nRT_h \ln\left(\frac{V_1}{V_2}\right)} = 1 - \frac{T_l}{T_h} \quad (15)$$

The efficiency thus depends on the temperature of the heating and cooling sources. A bigger temperature difference increases the theoretical efficiency limit. However, these temperatures are measured in Kelvin. Normal room temperature (20 C, 68 F) is 293.15 K. To have a theoretical efficiency of 50 %, a carnot engine cooled at room temperature must be heated at  $2 \cdot 293.15 = 586.3$  K; equivalent to 313.15 C or 595 F. Power plants burn their fuel at approximately this temperature and thus have a Carnot limit around 50%.

Internal combustion (IC) engines in cars are examples of cyclic heat engines. However, unlike in carnot engines, the heat in internal combustion engines comes from burning inside the chamber instead of an external heat source. See figure 7 [7].

Despite this difference, IC engines are still subject to the same Carnot efficiency limit. Today's IC engines burn their fuel at relatively low temperatures and have a Carnot limit of approximately 37% [8]. In reality, only about a quarter of the energy in the fuel burned in a car is used to move the wheels forward [9]. Furthermore, there is not hope for much improvement as we are already closing in on the Carnot limit.

Diesel engines are actually somewhat more efficient than gasoline ones. They burn fuel at a higher temperature and have higher compression ratios. This is why vehicles with heavy useage, such as trucks and buses, tend to have diesel engines [10]. Even though diesel beats gasoline when it comes to efficiency, electric cars are miles ahead. Almost 80 % of the energy taken from the grid is used to move the wheels. [9].

How do we know that the Carnot engine is the most efficient possible? Internal combustion engines are examples of cyclical heat engines with a slightly different design. Perhaps there could be another design out there that is more efficient than the Carnot engine? We will show that the existence of such an engine would violate the second law of thermodynamics.

First, note that the Carnot cycle is reversible. In the language of thermodynamics, this corresponds to  $\Delta S = 0$  for every full cycle. In a simple gas such as this one,  $S$  is a function only of  $P$  and  $V$ . This means a point on the PV diagram always has the same value of  $S$ . Intuitively, there is nothing stopping us from keeping the temperature constant during expansion using the colder source and give heat away to the hotter source during compression. This would however require external work done. More energy is needed to compress hot gas while

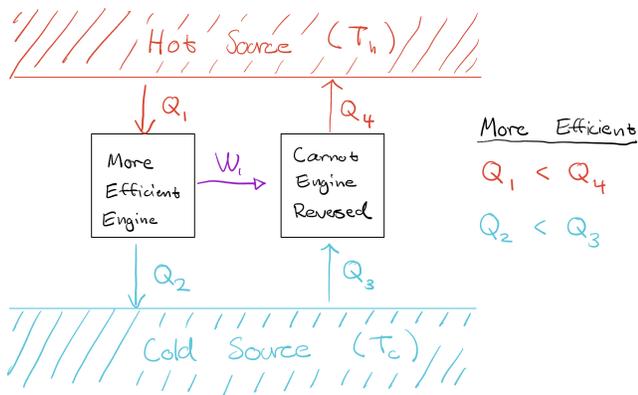


FIG. 6. Illustration of more efficient heat engine running from the same sources as the Carnot engine. We can use the work from the more efficient engine to run the Carnot engine in reverse, indefinitely cooling the cool source and heating the hot source. This contradicts the second law of thermodynamics.

keeping temperature constant than is gained from letting cool gas expand at constant temperature. The external work needed to make this happen is exactly the same amount as is produced by running the Carnot cycle in the "forward" direction. This can be seen from the way we derived the limit above: running the cycle in reverse corresponds to switching the sign of all integrals, leading to the same amount of negative work.

Let's imagine there is a more efficient cyclic heat engine. We could run this engine and the reversed Carnot engine using the same temperature reservoirs, as shown in figure 6. Our more efficient engine on the left takes heat energy  $Q_1$  from the hot reservoir and produces work  $W$ . The energy  $Q_2$  that cannot be used to do work is dumped into the cold reservoir. Whatever work  $W$  is produced from the more efficient engine can be used to run the reversed Carnot engine. The reversed Carnot engine uses this work to take heat  $Q_3$  from the cold source and output heat  $Q_4$  into the hot source.  $Q_3$  and  $Q_4$  have exactly the same value as for a regular Carnot engine producing work  $W$ . A heat engine being more efficient corresponds to producing more work  $W$  for an heat input  $Q$ . Since the left engine is assumed to be more efficient than a Carnot engine, this means it uses less heat to produce the same work:  $Q_1 < Q_4$ . More energy being used for work means there is less left to dump into the cold reservoir, and hence  $Q_2 < Q_3$  as well. The system in figure 6 thus creates a net energy movement from the cold source to the hot source without any external work being done to it. This decreases the entropy of the universe, violating the second law of thermodynamics. Hence such a system cannot exist, showing that the imagined more efficient engine is impossible to construct and that the Carnot engine is indeed the most efficient [11].

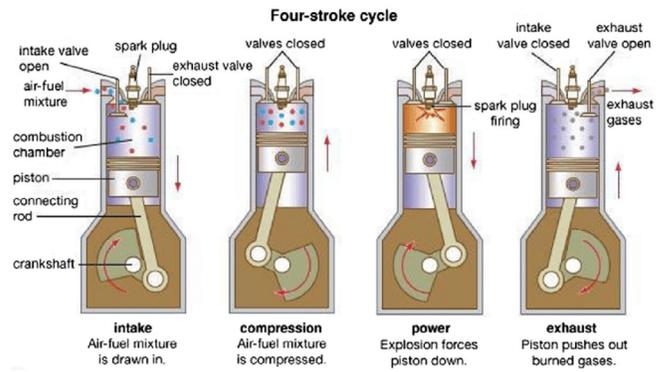


FIG. 7. Schematic of a four-stroke internal combustion engine. Most cars use this type of engine, while chainsaws and other simpler devices use the less efficient but mechanically simpler two-stroke design. Spark plug firing in the power phase only happens in gasoline engines; in diesel engines, the heat from the compression alone is enough to light the fuel. Figure is from [7].

## V. NUCLEAR POWER - FISSION

Nuclear power comes in two flavors: fission and fusion. All commercial power plants today are based on fission. In nuclear fission, large nuclei are split into smaller ones. Nuclear reactions generate around 1,000,000 times more energy per unit mass than chemical reactions [12]. The energy comes from the positively charged protons being further apart in the reactants than in the products. Originally, the positive protons were pushed together by colliding neutron stars [13]. Fission energy hence also originates in the stars, albeit not in the sun.

Uranium 235 is the isotope used in most fission power plants. When  $^{235}\text{U}$  absorbs a neutron, it quickly decays to Barium and Krypton while releasing 3 more neutrons and 207 MeV of energy, as shown in figure 9. If there is a sufficient concentration of other  $^{235}\text{U}$  nuclei around, the released neutrons will hit more than 1 other  $^{235}\text{U}$  nuclei on average, causing exponential growth in the energy released. If not moderated, this chain reaction will quickly release enough energy to demolish an entire city, as witnessed by the inhabitants of Hiroshima and Nagasaki in August 1945. Fission power plants use various materials and designs to absorb excess neutrons and keep the reaction in equilibrium. Energy is released from the reaction as kinetic energy in the products. The kinetic energy can be transferred to water molecules by putting water in proximity to the process. Water molecules with high kinetic energy, also known as steam, are directed through turbines to spin them around and generate electric power via Faraday's Law, as shown in figure 8 [14].

Fission power plants have had multiple accidents. The most famous ones are Fukushima in 2011, Chernobyl in 1986, and Three Mile Island in 1979. Maintaining the neutron balance is a delicate task and sometimes

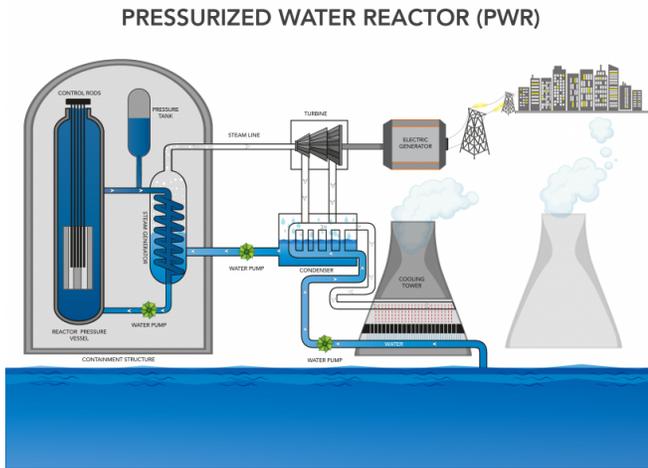


FIG. 8. Schematic of nuclear reactor. This type of reactor is called a pressurized water reactor. More than 65% of reactors in the United States are of this type. Every type of reactor utilizes the same fission reaction, but they differ in how they absorb excess neutrons and cool the core. Image from U.S. Department of Energy.

design flaws or extreme weather conditions take the system out of equilibrium. The consequences are catastrophic. Radioactive materials penetrate our skin with their energetic particles and wreck havoc in our cells. Moderate radiation levels increase the risk of cancer due to mutations forming when energetic particles hit our DNA. High radiation levels can kill on the spot. Radioactive substances can also travel long distances with wind and water. The Chernobyl disaster was first discovered by triggering radiation alarms at the Swedish fission plant Forsmark, 1,100 km away [15]. Radioactive emissions are however the only potential pollutant from the fission process. There is no carbon dioxide or monoxide, no nitrogen or sulfur oxides, and no ozone or particulate matter.

Even without accidents, fission plants still pollute with radioactive waste products. When the splitting of  $^{235}\text{U}$  is complete, the products are still radioactive. To avoid them causing human harm, some elements must be stored for hundreds of thousands of years [16].

There are lots of solutions out there, with consensus forming for deep geological storage as best-in-class [17]. Deep geological storage means covering the radioactive waste with different coatings and burying it several hundred meters below ground. Storing the waste in this manner mitigates the risk for immediate human harm. However, the enormous timescales handling fission waste cause concerns with geological stability and communicating the risk to future generation.

As a result of these problems, fission has lost public popularity. It's also prohibitively expensive to build new plants because of all the necessary safety measures. Furthermore, fission does not work great in tandem with

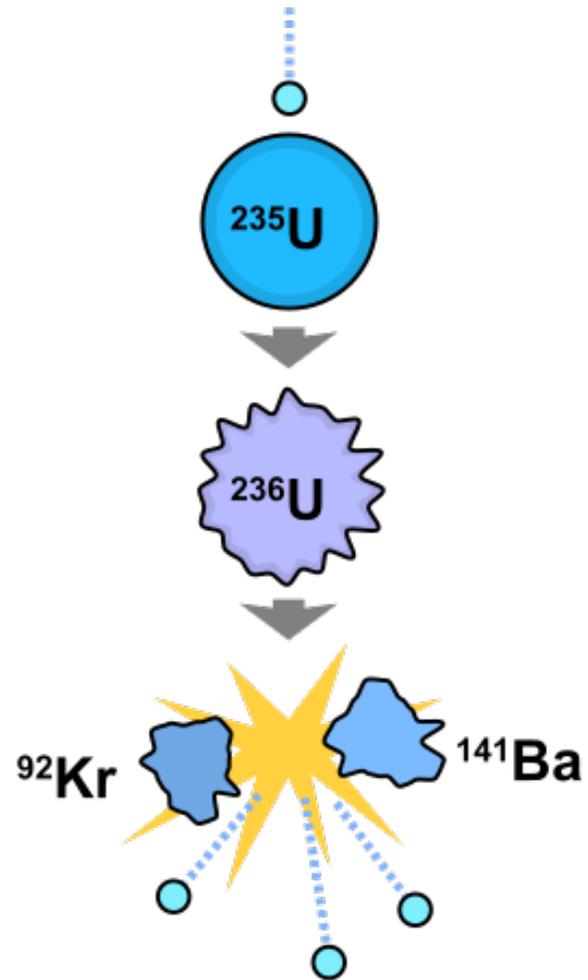


FIG. 9. Schematic of nuclear fission. The blue circles correspond to neutrons.  $^{235}\text{U}$  first absorbs a neutron to become  $^{236}\text{U}$ . This unstable isotope then decays, releasing 3 more neutrons to continue the chain reaction.

renewables. The main problem with wind and solar is intermittancy. Wind and sunshine vary tremendously on many different timescales. To provide power 24/7, an auxilliary plant thus has to react quickly to changing weather conditions. Modern fission plants take 2 hours to ramp up and down between 100% and 50% power [18]. Some of the variation in wind and solar, especially solar, happens on this time scale. But additional auxilliary energy production would be necessary to guarantee constant supply.

There are methods to mitigate the variability in renewable energy output, and our ability to predict the weather is increasing every day, but the two power sources will never be ideal companions. Fission power is therefore unlikely to expand significantly in the coming decades, despite being relatively clean and very reliable.

## VI. NUCLEAR POWER - FUSION

Fusion also leverages the energy of atoms, but in the opposite way. Instead of splitting large atoms into smaller atoms, small atoms are fused into larger ones. Fusion energy comes from an interplay of two forces: electrostatic repulsion between protons (Coulomb force) and the nuclear force between protons and neutrons. Electrostatic repulsion follows an inverse square law and can be felt over relatively long distances. The nuclear force decays exponentially with distances, but creates powerful attraction when nucleotides are about of  $10^{-15}$  m apart. The nuclear force is strong enough to overcome the Coulomb force at that distance, enabling the release of energy as nucleotides fall down a potential hill. Fusion is thus the only form of energy that does not originate in the stars. Instead, it is a form of energy copied from the stars.

Elements with fewer protons in their core feel less Coulomb force and are thus easier to bring together. Hydrogen fuels the fusion reactions in the sun, and our planetary attempts also revolve around the lightest element in the universe. See figure 12. When elements get more protons in their cores, nucleotides are too far apart to feel the nuclear attraction to each other, but they still feel the Coulomb repulsion. Fusion only generates net energy for elements light enough that less force is required to overcome the Coulomb repulsion than what is generated from nuclear attraction. Fusing nuclei heavier than iron-56 or nickel-62 will generally not generate net energy [19].

In order to overcome the repulsive Coulomb force, even one proton hydrogen nuclei need to crash into each other extremely fast. Fast moving nuclei means high temperatures. Our sun fuses nuclei together in its core at temperatures around 15,000,000 K. To create fusion here on earth temperatures of approximately 200,000,000 K are required [20].

This temperature is high enough to strip electrons from their nuclei, creating a soup of charged particles known as plasma. Achieving and maintaining these high temperatures are a major technical obstacle for fusion energy. No material we know of can withstand the temperatures required. Current attempts use magnetic fields to keep the charged plasma away from anything solid. The strongest magnet ever was developed as a part of fusion plasma confinement [21].

Current attempts place strong superconducting magnets in a torus-like shape, as shown in figure 10 [22]. The plasma will be confined to the center of the torus by the magnetic field. A machine that use magnetic fields to confine plasma in a torus shape and initiate the fusion reaction is known as a Tokamak. Another approach using what is known as a stellarator, shown in figure 11. Stellarators have the advantage of requiring less energy input into the plasma, but it comes at the expense of a more complex design [23].

No experiment has yet generated net power from

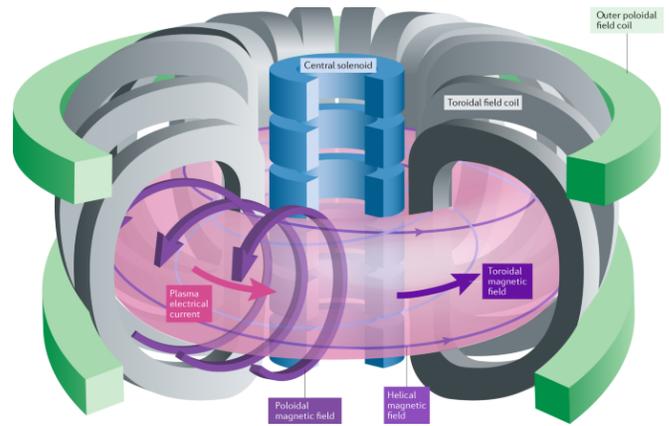


FIG. 10. Tokamak in torus shape. Magnets are found along the edges of the torus and in the center. The field is from the magnets is shaped to confine the plasma to the center of the torus, far away from any meltable materials [22].

fusion. Fusion has been known for decades but all efforts to control the process thus far have been futile. It has therefore been characterized as the energy source that is "always 20 years away". However, recent advances are cause for hope. High temperature superconducting magnets and 3D printing are novel technologies that promise better control of plasma. Improved computational resources also help create more accurate prediction and control of the phenomena. Private funding has flowed into the fusion space in recent years. Cambridge based MIT-spinoff Commonwealth Fusion Systems was founded in 2018 and raised \$1.8 billion in their latest funding round. In southern France, a multinational government funded effort called ITER (International Thermonuclear Experimental Reactor) is attempting to create the first net positive fusion reaction. Some startups say they will put fusion power on the grid by 2025 whereas state-funded efforts tend to have more modest timelines: ITER aims at a full fusion reaction by 2035 [24, 25].

Despite not being commercially available for a while, fusion presents a clean and almost infinite supply of energy. The fuel, hydrogen, can come from water. One glass of water can provide enough energy for a human lifetime [24]. In addition, there are no problems of intermittency like with wind or solar. If fusion can be technically achieved and made economically viable, it would be a fantastic energy source. Technical barriers and costs are huge barriers to overcome, but the almost infinite supply of safe and clean energy on the other side would make it well worth the effort.

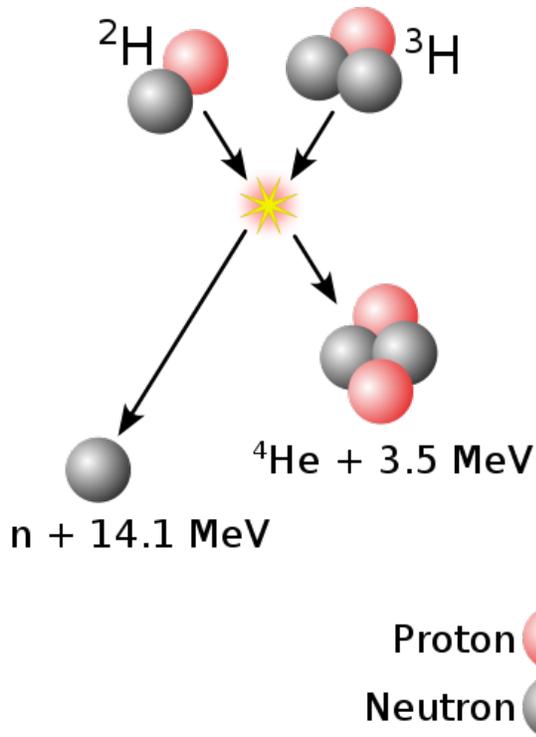


FIG. 12. Fusion of deuterium and tritium into helium and a neutron. Energy is released as kinetic energy of the particles.



FIG. 11. Two versions of a stellarator. These devices confine plasma through the use of magnetic fields, but without running a current through the plasma itself. They enable lower power input to the plasma, but are more complex to design [23].

## VII. WIND POWER

Wind power is actually a form of solar power. The wind blows because the sun heats up our planet. And it can be really powerful. In the US, land based 100-meter hub wind power operating at 30% capacity has the potential to provide 44 trillion kWh each year more than 10 times the current US electricity consumption [26, 27].

Wind power currently provides about 380 billion kWh of energy each year - 9.2 % of total U.S. consumption [28]. This is less than 1% of the energy available. Denmark produced 16 billion kWh, 48% of its electricity, from wind in 2020 [29].

Wind is the primary form of renewable energy right now, supplying about 50% more energy than hydropower and more than 3 times as much as solar [30]. The most powerful wind turbine today is capable of generating 14MW of power. It features a 220-meter rotor diameter and has a height of 260 meters. A single rotation can power a home for two days [31]. To visualize the size of this monstrous machine, think of two football fields spinning around a pivot 150 meters in the air. The component sizes exceed infrastructure limitations and the turbine can thus only be deployed at sea. Modern wind turbines are truly engineering miracles.

### Energy available in wind

Let's try to find the potential amount of power a wind turbine can generate. First, recall the formula for the kinetic energy  $KE$  of an object with mass  $m$  moving with velocity  $v$ :

$$KE = \frac{1}{2}mv^2 \quad (16)$$

This applies to every individual molecule in the air. We can change the mass  $m$  to density  $\rho$  times a volume of air  $V$  to get the expression for the kinetic energy in a volume  $V$  of air:

$$KE = \frac{1}{2}\rho Vv^2 \quad (17)$$

To move from the kinetic energy of the wind to the power output of a wind turbine, we must find the volume of wind that blows through the turbine in a given period of time. If the turbine has area  $A$  and wind velocity  $v$ , the volume passing through the turbine in a time  $t$  is given by  $Avt$ , under the maximizing assumption that the wind blows straight through the turbine. Plug this in for  $V$  in equation 17 to get the energy available in  $t$  units of time.

$$E = \frac{1}{2}\rho Av^3t \quad (18)$$

Moving from energy to the power is as simple as taking a time derivative:

$$P = \frac{1}{2}\rho Av^3 \quad (19)$$

There are thus two factors that determine the power output of a wind turbine: area and wind speed.

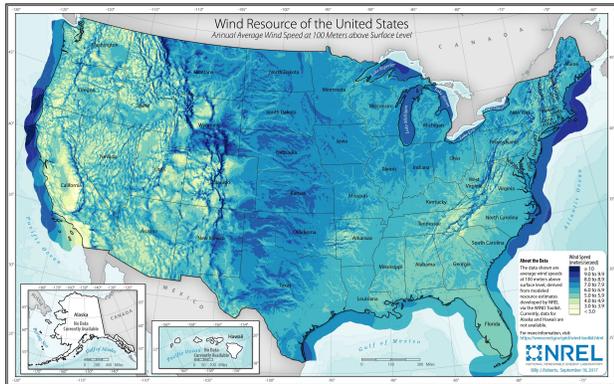


FIG. 13. U.S. Wind Speeds at 100m elevation. The strongest winds are found off the coast in the northeast and northern California. These areas are close to major population centers on the coasts, but despite the transportation savings and strong winds offshore wind turbines have a higher LOCE than onshore. Offshore wind also requires sea depths less than 50m. Map is from U.S. Department of Energy [33].

First, power is directly proportional to the area covered by the turbine. The circular area swept out by the blades is  $\pi r^2$ , where  $r$  is the length of the rotor blade. The potential power output of a turbine is thus proportional to the *square* of the rotor blade length. The materials involved, the weight of the blade, and other input costs grow (approximately) linearly. This means that longer blades gives more power per unit of input. The current bottleneck for the length of turbine blades is actually the infrastructure (tunnels, bridges, etc) to transport them.

Second, power is proportional to the *cube* of the wind speed. This means you really, really want to put your turbine where there is a high average wind speed. Twice the average wind speed gives eight times the power for the same turbine. In economic terms, this means the return on investment is eight times larger. Average wind speeds also increase with elevation. One can estimate the wind speed  $v$  at height  $h$  through measuring speed  $v_0$  at reference height  $h_0$  and applying the following function:

$$v(h) = v_0 \cdot \left(\frac{h}{h_0}\right)^\alpha \quad (20)$$

The coefficient  $\alpha$  is called the Hellman coefficient and typically takes on values in the range 1/10 to 1/4 [32]. However, since wind power depends on the cube of the wind speed, the influence of height on power becomes much larger.

Elevating turbines to reach higher wind speeds is expensive as it requires larger towers and more complex assembly, but can be a strategy for increased power output. Building offshore wind turbines is significantly more expensive than onshore, see figure 2, but can make economic sense quickly if average wind speeds

are somewhat higher. In addition, offshore wind is not subject to the same infrastructure restrictions as onshore wind and can thus support longer blades and higher towers, further increasing turbine power output. Traditional offshore wind turbines are anchored to the sea floor and thus limited to shallow water, but there are floating wind turbines under development. If the cost is not too high, floating wind would enable tapping into even more energy resources.

### A. Theoretical turbine efficiency limit

For the energy contained in the wind to be useful to us humans, turbines need to convert the kinetic energy of the air into electricity. It turns out there is a theoretical limitation on how efficient this conversion can be. The limitation is called the "Betz Law" and comes from the fact that the wind has to keep moving after it passes through the turbine. Air particles stopping behind the turbine would quickly violate the conservation of mass, as more mass would flow into the turbine than out of it. Hence the air has to keep moving with some velocity. What fraction of the velocity before the turbine should remain after the turbine for optimal efficiency?

On a first glance, one might think that a fraction closer to 0 would always be better. But there is a tradeoff: the slower the wind goes after the turbine, the slower the wind will have to move across the turbine as well. Acceleration cannot happen instantaneously. We already know the cubic dependence of turbine power output on wind speed, so there is no surprise having a higher velocity across the turbine can make up for extracting a smaller fraction of the energy available in wind with that velocity.

To derive the Betz limit, we will make 3 simplifications of the real world situation:

- Ignore the rotor hub and any blade drag. Taking this into account would only decrease maximum efficiency.
- Assume no wind is blowing parallel to the turbine. Any parallel wind would also only decrease maximum efficiency at a given wind speed.
- Assume air density is constant. This is necessary as the solution builds on conservation of mass.

The derivation builds on mass flow  $\dot{m}$ . The mass flow through an area  $A$  by air with density  $\rho$  can be expressed as  $\dot{m} = A\rho v$  where  $v$  is the flow velocity component perpendicular to the area  $A$ . To avoid build-up of mass anywhere, which would be unphysical, the same amount of mass must flow through the entire setup. If we let  $A_1$  and  $v_1$  denote the area and velocity before the turbine,  $A$  and  $v$  the area and velocity at the turbine, and  $A_2$  and  $v_2$  the area after and velocity the turbine (see figure 14), we get that:

$$\dot{m} = A_1\rho v_1 = A\rho v = A_2\rho v_2 \quad (21)$$

Now we need to find the power in the turbine. The kinetic energy given by an air molecule with mass  $m_a$  as

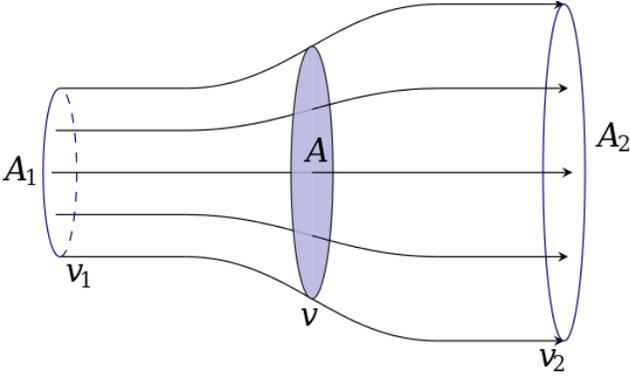


FIG. 14. Setup for Betz Law derivation. The larger area after passing the turbine is a consequence of mass flow continuity combined with lower wind velocity. Image from Wikipedia.

to the turbine can be written as:

$$\Delta KE_a = \frac{1}{2} m_a (v_1^2 - v_2^2) \quad (22)$$

Summing over all molecules crossing the turbine and taking a time derivative allows us to substitute  $m_a$  for  $\dot{m}$  and gives an expression for the power transmitted to the turbine:

$$P = \frac{1}{2} \dot{m} (v_1^2 - v_2^2) = \frac{1}{2} \rho A v (v_1^2 - v_2^2) \quad (23)$$

We can also find the power by looking at the force from the air on the turbine. Newton's 2<sup>nd</sup> law states that  $F = ma$ . We don't know the instantaneous acceleration at any point, but we know the total change in velocity as the air passes through the turbine. We also know how much mass changes velocity in a given unit of time. The force can be expressed as a product of these quantities.

$$F = \dot{m} \Delta v = \rho A v (v_1 - v_2) \quad (24)$$

Moving from force to power is done by taking the velocity with which the force is acting with into account.

$$P = Fv = \rho A v^2 (v_1 - v_2) \quad (25)$$

We can now equate our two expressions for the power of the turbine:

$$\rho A v^2 (v_1 - v_2) = \frac{1}{2} \rho A v (v_1^2 - v_2^2) \quad (26)$$

The assumption of constant density and (trivially) non-zero area allows us to simplify the above expression:

$$v = \frac{1}{2} (v_1 + v_2) \quad (27)$$

We can now use this relation to eliminate  $v$  from our

power expression and maximize based on the  $v_1/v_2$  ratio:

$$\begin{aligned} P &= \frac{1}{2} \rho A v (v_1^2 - v_2^2) = \\ &= \frac{1}{4} \rho A (v_1 + v_2)(v_1 + v_2)(v_1 - v_2) = \\ &= \frac{1}{4} \rho A (v_1^3 + 2v_1^2 v_2 + v_1 v_2^2 - v_1^2 v_2 - 2v_1 v_2^2 - v_2^3) = \\ &= \frac{1}{4} \rho A v_1^3 \left( 1 + \frac{v_2}{v_1} - \left( \frac{v_2}{v_1} \right)^2 - \left( \frac{v_2}{v_1} \right)^3 \right) \end{aligned}$$

Letting  $x = v_2/v_1$ , differentiating with respect to  $x$  and setting equal to 0 gives us:

$$\frac{dP}{dx} = 0 \implies \quad (28)$$

$$\frac{1}{4} \rho A v_1^3 (1 - 2x - 3x^2) = 0 \implies \quad (29)$$

$$-\frac{1}{4} \rho A v_1^3 (3x - 1)(x + 1) = 0 \implies \quad (30)$$

$$x_1 = \frac{1}{3}, x_2 = -1 \quad (31)$$

The only physical solution is the positive one; the wind must go in the same direction before and after the turbine. Hence the optimal ratio is 1/3. Plugging in this ratio into the expression for power results in:

$$P = \frac{1}{4} \rho A v_1^3 \left( 1 + \frac{1}{3} - \frac{1}{9} - \frac{1}{27} \right) = \frac{8}{27} \rho A v_1^3 \quad (32)$$

The power available in the wind itself was given by  $\frac{1}{2} \rho A v^3$ . We are interested in what fraction we can theoretically harvest with a turbine. Let's call this the efficiency. It's given by:

$$\text{Efficiency} = \frac{\frac{8}{27} \rho A v_1^3}{\frac{1}{2} \rho A v_1^3} = \frac{16}{27} = 0.5925 \approx 59\% \quad (33)$$

It is thus theoretically to design a turbine that harvests more than 59% of the power available in the wind. Today's wind turbines are already closing in on that limit: most turbines extract 50% of the power from the wind that passes through them [34]. This means there are not a lot of room for efficiency improvements: we simply need to put more big turbines in windy locations.

## VIII. ALTERNATIVE WIND POWER

### AeroMINE Technologies

Startup AeroMINE Technologies founded in 2020 have developed a new type of wind turbine shown in figure 15. Their big selling point is a design without exposed moving parts. They claim this makes the turbine safer, quieter, and cheaper to maintain. While these features are always important, they matter less for utility-scale production far from populated areas. AeroMINE,

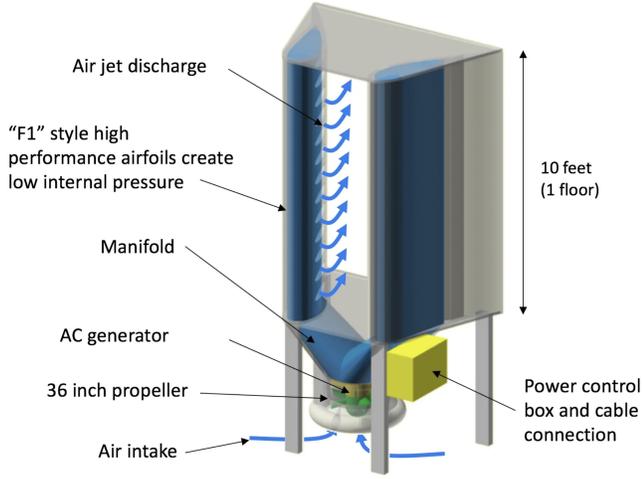


FIG. 15. AeroMINE wind turbine design. The 36 inch propeller is powered by the pressure difference created when wind blows across the airfoils.

however, is not trying to replace traditional wind turbines in utility-scale production. They are targeting the distributed wind market.

Let's understand how their system works. Much like a regular wind turbines work like fans in reverse, AeroMINE's system works as a "bladeless" fan in reverse. A "bladeless" fan is not actually bladeless, but instead hides the blade underneath the air stream. AeroMINE hides the blade spinning the generator underneath the opening where the wind blows through. The opening of the turbine has an airfoil on each side. An airfoil is any object that generates a pressure difference when a gas or fluid moves over it. These airfoils are designed to lower the pressure between them, as shown in figure 16 [35].

Air will then flow from the air intake, where the pressure is atmospheric, to the space between the airfoils, where the pressure is lower. The blue sections of figure 15 are hollow and connected, allowing air to enter through the air intake, pass the propeller, and exit through the air jet discharge holes into the space between the airfoils. A pressure difference  $\Delta p$  across the air jet discharge holes is able to generate a power  $P$  given by:

$$P = \Delta p \cdot V_{jet} \quad (34)$$

where  $V_{jet}$  is the volume flow through the air jets. The volume flow through the jets can in turn be expressed as:

$$V_{jet} = u_{jet} \cdot A_{jet} \quad (35)$$

where  $u_{jet}$  is the velocity of the flow through the jets and  $A_{jet}$  is the total area of all the jets. Bernoulli's equation for incompressible flow states that the velocity  $u_{jet}$  can be related to  $\Delta p$  through:

$$\Delta p = \frac{\rho}{2\gamma} u_{jet}^2 \quad (36)$$

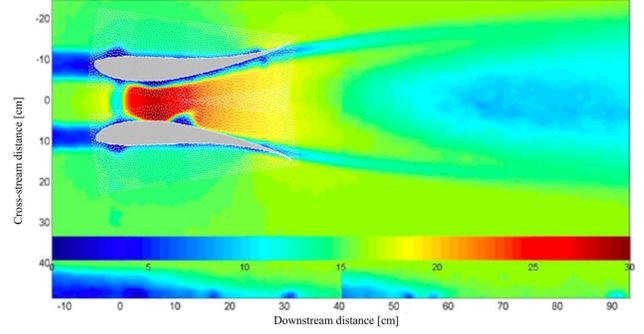


FIG. 16. Velocity map for AeroMINE prototype in 15 m/s wind. The scale on the bottom marks local wind speed in m/s. Higher wind speed corresponds to lower pressure by the Bernoulli equation.

where  $\rho$  is the density of the air, which is assumed to be constant in application of this equation, and  $\gamma$  represents losses in the process of discharging the jets. We can use the same Bernoulli equation to relate the pressure drop to the free stream velocity  $U$ , only this time we have include a coefficient  $C$  related to the choice of airfoils:

$$\Delta p = \frac{\rho}{2} \cdot C \cdot U^2 \quad (37)$$

Substituting into 34 using 36 and 35, we get:

$$P = \frac{\rho}{2} \cdot C \cdot U^2 \cdot u_{jet} \cdot A_{jet} \quad (38)$$

We can use 36 and 37 to replace  $u_{jet}$  with  $U$  as:

$$P = \frac{\rho}{2} C^{3/2} U^3 \gamma^{1/2} A_{jet} \quad (39)$$

The power available in the area covered by AeroMINE's design is given as the power in the wind flowing through the largest area  $A_{exit}$ . This is the area at the back in figure 15. We saw earlier that the wind power blowing through  $A_{exit}$  with velocity  $U$  is  $\frac{1}{2}\rho U^3 A_{exit}$ . The efficiency  $\eta$  of the AeroMINE design is thus given as:

$$\eta = C^{3/2} \gamma^{1/2} \frac{A_{jet}}{A_{exit}} \quad (40)$$

This derivation followed largely the one given in the paper describing the AeroMINE device [35]. In the paper, they claim that individual testing of these parameters indicate an efficiency of 51 % is achievable. This would be on par with normal turbines and close to the Betz limit. So far, however, the maximum efficiency achieved by the team at AeroMINE is 25 % [36].

But we already have wind turbines. Even if it is true that their design is safer, quieter, and cheaper to maintain compared to traditional turbines, which might very well be the case, will it be able to compete on cost? To make rooftop wind a viable alternative, it has to have a LCoE similar to rooftop solar. In their paper,

Westergaard et al. present a calculation of the LCoE for AeroMINE. Their calculation is based on current rooftop solar installation costs and optimum AeroMINE performance. They claim the AeroMINE system can reach a LCoE of 10 ¢/kWh with an average wind speed of 5 m/s [35]. For comparison, this is lower than the 11 ¢/kWh LCoE of residential PV solar systems installed in 2022 [37].

I have some doubts about the LCoE reached by AeroMINE because they don't specify any of the assumptions they make in reaching it. First, it's unclear what their statement about optimal performance means. Does that refer to the 25% efficiency they achieved in testing or the 50% they think is theoretically possible? What directional variation did the wind have in their model? Did they use rotating devices or stationary? What values did they use for discount rate, interest rate, and annual maintenance cost? How do they know the price of manufacturing the device since they only have prototypes so far? A report attempting to seriously compute the LCoE would specify all these parameters and make clear why the assumptions made are reasonable. There should also be confidence intervals around each assumption and an interval of potential LCoE values. The lack of specificity makes me believe they are hiding some unreasonable assumptions that would bring up the LCoE way past the one of solar.

In addition, the cubic scaling of power with wind speed and quadratic scaling with area swept already puts distributed wind at a disadvantage compared to distributed solar. Furthermore, wind in populated areas is far less predictable and turbulent than in open areas or over the ocean. This reduces the efficiency of distributed wind further.

Even with a higher LCoE than rooftop solar, there might still be a use for the AeroMINE turbines. Some places are rich in wind resources and poor in solar. However, regular wind turbines would still be able to provide power at a lower cost. The claimed advantage for the AeroMINE design is more safety and less maintenance. There is no evidence for the claim that the AeroMINE design requires less maintenance as it is still a prototype, and it's unclear to me how much of priority safety would be for something already situated on a rooftop.

AeroMINE also imagines their turbine creating value in combination with solar panels. Combining power sources reduces fluctuations and enables a higher average output. Furthermore, solar has the disadvantage of only producing power during the day. AeroMINE turbines can generate electricity even after the sun has set. However, regular wind turbines would provide the same advantage at a lower cost here too. If it's a roof where lots of people walk around and risk getting hit by turbines, or AeroMINE actually requires less maintenance and the building is situated in a very remote location, I can imagine their solution having an advantage over traditional wind.

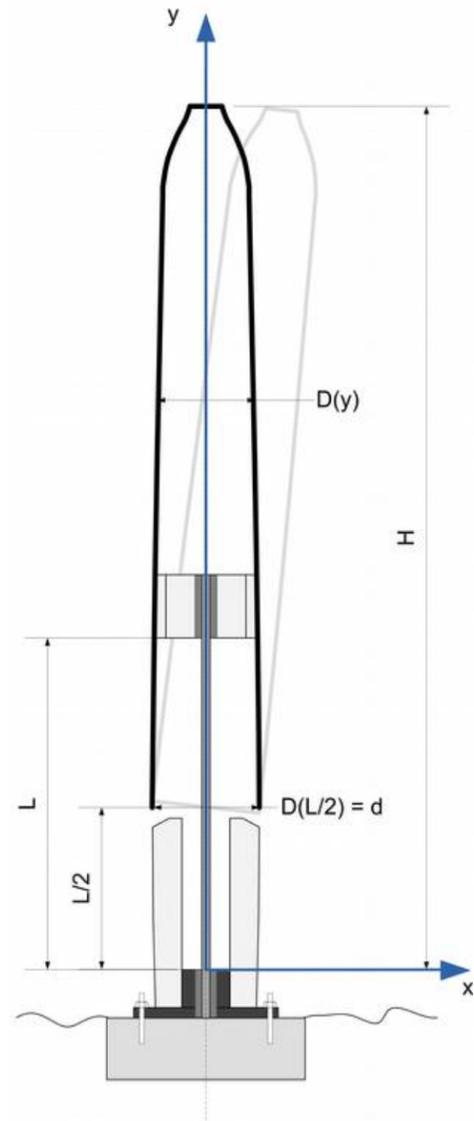


FIG. 17. Vortex bladeless design. The oscillations will move a magnet through a wire in the middle of a structure to generate electricity.

Overall, I'm skeptical AeroMINE has a design good enough to replace regular wind turbines in any scenario.

### Vortex Bladeless

Spanish startup Vortex Bladeless founded in 2014 has developed a novel way of generating electricity from wind. Their design cannot even be called a "wind turbine" in the strict sense of the word, because a turbine is something that revolves around an axis. Vortex bladeless produces power by oscillations instead of revolutions.

Their design is shown in figure 17 and based on an aerodynamic phenomenon called vortex shedding. Vortex

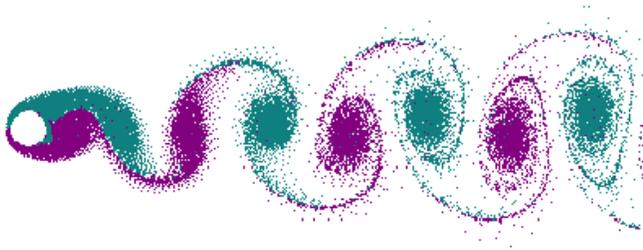


FIG. 18. Illustration of vortex shedding. Low pressure is created on alternating sides of the cylinder, causing it to oscillate.

shedding occurs when a fluid flows past a bluff body at certain velocities. A bluff body has a technical aerodynamical definition, but we can think of it as the opposite of streamlined. The cylindrical shape used by Vortex bladeless certainly qualifies, and we will limit ourselves to the case of cylinders from now on. In figure 18, vortex shedding of a cylindrical body is shown. As can be seen from figure 18, vortex shedding is a periodic phenomena. Vortices come of the cylindrical shape with a characteristic frequency  $f$  given by:

$$f = \frac{St \cdot V}{D} \quad (41)$$

where  $St$  is the so-called Strouhal number,  $V$  is the flow velocity, and  $D$  is the diameter of the cylinder. The Strouhal number is commonly taken to be a constant under regular wind conditions.

A cylinder that is not rigidly attached can begin to oscillate due to the periodic pressure difference caused by the vortices. If the frequency of vortices matches the resonance frequency of the cylinder, the oscillations will become significant. However, since the frequency depends on the velocity of the wind, and we know from before that the velocity depends on height above ground, the frequency will be different at different heights of a cylinder. Vortex bladeless plans to tackle this problem through the other variable in equation 41: cylinder diameter. By increasing the diameter with height, Vortex bladeless thinks they can get the same resonance frequency for the entire structure.

With the same resonance frequency in the entire structure, we can consider the area swept. The angular displacement in their tests is only about  $5^\circ$  in each direction [38, 39]. A vortex bladeless where the oscillating part has length  $R$  thus covers an area  $A_v$  given by:

$$A_v = 2 \cdot \left( \frac{5}{360} \right) \cdot 2\pi R^2 + TR \quad (42)$$

where we approximate the  $5^\circ$  angular displacement as sweeping a circular arc of radius  $R$  in each direction

and  $T$  is the thickness of the pole. The thickness of the pole varies with  $R$  in Vortex bladeless's design, but the product of  $T$  and  $R$  can safely be taken to be about the same size as the area swept. A regular wind turbine with blade length  $R$ , which can be achieved with approximately the same tower height as the vortex bladeless height, will sweep an area  $A_r = 2\pi R^2$ . The ratio between the two is thus:

$$\frac{A_v}{A_r} = \frac{20}{360} = \frac{1}{18} \approx 6\% \quad (43)$$

On their website, they state that the area swept is 35% of a traditional turbine [40]. Their number comes from comparing their tower height with the maximum height reached by a rotor blade of a traditional turbine. This would increase the ratio by a factor of 4. Adding in slightly more of the vortex bladeless height being used for energy generation can explain the discrepancy between the 6% and the 35% on their website. I'm not a wind power expert so I don't know what the fair comparison to make is, but it's helpful to at least understand that their tower must be as tall as an entire regular turbine to produce a significant fraction of the power.

In addition, the lack of rotation in the system requires the use of a linear generator. Vortex bladeless claims to have designed a linear generator that converts 70 % of the linear movement into energy. This is however still less than the 80 - 90 % conversion efficiencies of regular generators [41].

If each of the three rotors and the main tower all cost as much as an entire vortex-bladeless system, the traditional turbine will cost four times as much. But this traditional turbine will generate more than 18 times the power because of the larger area swept and more efficient generator. Costs might be even more different due to a simpler manufacturing process and little material use in the top part of the Vortex bladeless design, but I have a hard time seeing this design beating traditional turbines in production cost per kWh.

On their website, they claim that their system only has 5 % of the maintenance cost of regular turbines. While there might be truth to this as their design is simpler than traditional turbines and the electrical parts are located closer to the ground, there are no commercially deployed systems and any statement about maintenance costs have to be taken with a grain of salt.

Vortex bladeless also state that land costs are only 70% for their system compared to traditional turbines [40]. I can't see how land costs would change depending on what turbine you place on it. Perhaps they are making a statement about how closely you can pack the turbines together, but that would require careful consideration of the fluid dynamics involved that I doubt they have gone through.

Vortex bladeless thinks of their generator as a both a distributed and utility-scale solution. In utility scale production, cost is king. Vortex bladeless claim their turbine will have as much as a 40 % lower LCoE than

traditional turbines [40, 41]. They don't specify exactly how they reached this number, but we can draw some clues from their website. If we use their calculation with same height and 35 % of swept area, the material costs are clearly higher as there is much more stuff per area covered. The savings must thus be on maintenance, manufacturing, and installation. Since their design is still a prototype, they really have no idea how expensive these things are going to be.

In addition, vortex bladeless generators will run into challenges from fluid mechanics as they increase to utility size. Sheila Widnall, aeronautics and astronautics professor at MIT, says that "when the cylinder gets very big and wind gets very high, you get a range of frequencies. You won't be able to get as much energy out of it as you want to because the oscillation is fundamentally turbulent" [41].

Infrastructure capabilities will also be a limiting factor. The top part of their turbine is larger than any part of a traditional turbine with the same height. It might thus be limited in size due to infrastructure limitations. This, in combination with the limitations from fluid dynamics, makes Vortex bladeless use land less efficiently. Traditional turbines can use vertical space not accessible to the Vortex bladeless, thus making better use of the available land. Winds at high altitudes are also stronger, further increasing the advantage for traditional turbines.

It's possible to make somewhat accurate cost estimates, but also easy to be overly optimistic about your technology when reality is far away. My bet is on the latter and that this technology is unlikely to produce utility scale wind power at a 40% lower LCoE.

In distributed production, Vortex bladeless won't run into the same scaling issues. Noise and space are also more important considerations. Their claims about silent operation are however not convincing to everyone. The same Sheila Widnall that pointed out the scaling issues also mention that "the oscillating frequencies that shake the cylinder will make noise. It will sound like a freight train coming through your wind farm [41]". The sound of freight trains is likely to put off some would-be distributed users. We must also remember that the scaling laws of windpower are fundamentally unfavorable for distributed production. In conclusion, distributed wind is unlikely to be a significant market.

If their cost estimates are correct and they manage to make their product utility-scale, this technology could become an important part of our future. There are however many issues left to resolve and questionmarks to straighten before we get there, and I fear the road to travel might be longer than Vortex bladeless are hoping for.

## IX. WATER POWER

The vast majority of water power comes from hydroelectric dams. The Three Gorges dam in China is

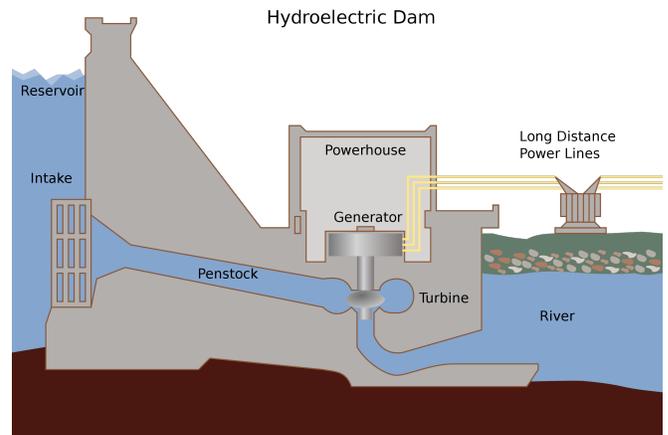


FIG. 19. Schematic of hydroelectric dam. The pressure of the water above the intake turns the turbine and generates electricity. Image from Wikimedia [45].

the largest in the world right now, capable of generating 22,500 MW of power [42]. Despite not always operating at full capacity, the dam managed to produce 112 TWh in 2020, a world record [43]. For comparison, the whole country of Sweden used 127 TWh that same year [44].

Dams work by converting gravitational potential energy of water to electric energy on the grid. Initially, the gravitational potential energy comes from the sun. Water vaporized by the sun's rays rises in the atmosphere to form clouds. The clouds then let the water fall down in the form of rain. Hydroelectric power is generated by letting the falling water spin strategically placed generators. A strategic place for a generator is where lots of falling water is naturally gathered and can run through it. We call these places rivers. The gravitational potential energy of  $m$  kilograms of water can be computed as  $mgh$ , where  $g$  is the gravitational acceleration at the surface of the earth and  $h$  is the height above the surface of the earth. A really good river for energy generation is thus one that can supply lots of water falling a long distance. More kilograms of water increases  $m$  in the energy calculation and a longer falling distance increases  $h$ .

Dams are an artificial construction to increase the control and energy output of rivers. A dam is essentially a big wall stopping a river, with a small opening to direct the river flow through power generators, as shown in figure 19 [45]. Dams increase control of the power generation by creating a reservoir of water behind them. River flow usually vary with the seasons. There is more water in spring as mountain snow is melting and less in the winter as downfall stays frozen. Reservoirs can absorb these seasonal variations and provide a stream of water through the power generators whenever necessary.

Dams also enable capturing more of the water's gravitational potential energy. To exit at the bottom, water essentially has to fall the entire height of the dam.

No water molecules *actually* falls all the way from the top to the bottom in one instance to generate power. The power at the bottom instead comes from the pressure provided by all the water weight on top of it. Pressure  $p$  at a depth  $d$  can be computed as  $p = \rho g d$ , where  $g$  is gravitational acceleration and  $\rho$  is the density of the fluid. If we look at a volume  $v$  of water at depth  $d$ , it contains  $p v = \rho g d v$  units of potential energy. This means there is a linear relation between the height of the damn and the energy that can be extracted from each unit of water. A higher damn thus not only allows storing more water, but enables extracting more energy from each unit of water. Intuitively, this corresponds to a higher dam capturing more of the gravitational potential energy as vaporized water makes its way down from the clouds.

To understand the total energy stored behind the dam, we can look at the potential energy of the water in the reservoir. The energy in the reservoir is computed as the sum of all the gravitational potential energy of the water in it. Since there is a continuum of heights, we have to make it an integral:

$$E = \int_0^d m g h \, dh = \frac{1}{2} m g d^2 \quad (44)$$

The energy stored in a damn thus grows as the square of the height. This quadratic dependence is the result of two linear ones: both the amount of water and the energy in each unit of water are linear with respect to the height of the dam. This is why high dams make a lot of sense. In reality, this dependence on height would be even stronger. The area of a slice of water at a given height almost certainly grows the higher up you go - rivers tend to flow in roughly triangle-shaped valleys. There are no theoretical efficiency limits to the process of converting the energy in the water to electricity and average efficiency is already around 90% [46].

## X. SOLAR POWER

Solar power refers to directly tapping the energy in the suns rays instead of going through the indermediaries of wind or water. Despite the sun being 150 million kilometers from earth and only transmitting 0.000000045% of it's energy to our planet, radiation from the sun blasts the earth with up to 1365 watts per square meter [47].

However, 1365 is the absolute peak, measured above the atmosphere and perpendicularly to the sun. To find the average available energy for a solar panel, one must take atmospheric absorbtion, the earth's rotation, and latitude into account. The earth's rotation means that any given panel only spends half its time facing the sun. In addition, most of the time when a panel is facing the sun, it's not at a perpendicular angle. A perfectly perpendicular angle is only achieved at noon in areas close to the equator. In the morning and evening, when the sun is lower in the sky, less solar power is delivered

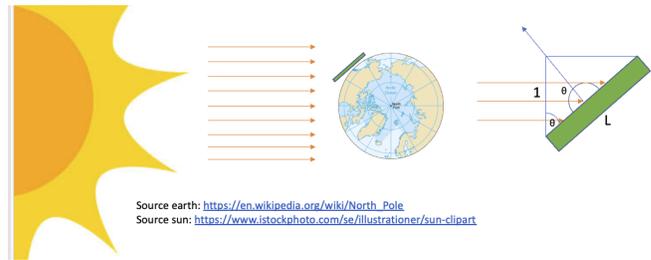


FIG. 20. Illustration of rays from the sun incident on the earth. The earth is shown looking down from the north pole. One counter-clockwise rotation happens every 24 hours. The green rectangle represents a part of the planet on which we are considering incident radiation. On the right hand side, you can see how the radiation spreads out over a larger land area if the angle to the sun is larger.

to any given area of land. See figure 20. When the sun's rays make an angle  $\theta$  with the earth normal, see figure 20, power delivered to the perpendicular area is spread over an area  $L$ . If the perpendicular is chosen to have length 1, we get the relation  $L \cos \theta = 1 \implies L = \frac{1}{\cos \theta}$ . At any given  $\theta$ , the power is divided by this factor. Actual power delivered at an angle  $\theta$  is thus perpendicular power multiplied by  $\cos \theta$ . To find the impact on average power, we take the average of  $\cos \theta$  for  $\theta \in (-\pi/2, \pi/2)$ .

$$\text{Avg. angle factor} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos \theta \, d\theta = \frac{2}{\pi} = 0.64 \quad (45)$$

A similar factor appears when considering latitude. In figure 20, the angle  $\theta$  with the normal could also have been created by standing at a higher latitude. If  $\alpha$  is the latitude angle, power delivered will be scaled down by  $\cos \alpha$ . Since the earth axis is tilted, the value of  $\alpha$  depends on the season. As the earth rotates the sun, the angle between the earth's axis of rotation and the sun changes. Late December in the northern hemisphere, at the winter solstice, the angle is  $23.5^\circ$ . At the summer solstice in late June, the angle is the same, but in the opposite direction. In Boston, latitude  $42.5^\circ$  N, rays from the sun have an angle of  $42.5 - 23.5 = 19$  degrees at the summer solstice and  $42.5 + 23.5 = 66$  at the winter solstice. Adjustable solar panels can adjust their angle to the sun and thus overcome the latitude factor, but stationary panel efficiency will be reduced by the seasonal angle variation, no matter their fixed angle and where on the planet they are placed. At the optimal panel angle, roughly the same as the latitude of the location[48], incoming ray angle  $\alpha$  will vary sinusoidally between  $-23.5^\circ$  and  $23.5^\circ$  compared to the panel normal. Since  $\cos 23.5 = 0.92$ , more than 90% of the power will be retained at any given time. Seasonal angle variation will thus not contribute significantly - only 4% of the power disappears on average through this effect.

Hence, when looking at a specific spot on the planet, assuming panels are angled by latitude, and taking both

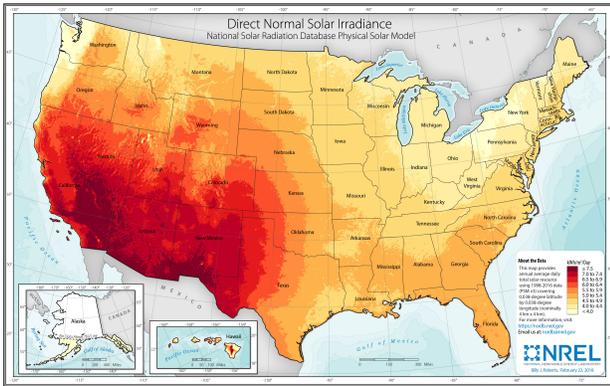


FIG. 21. Map of solar power available in the United States. The map shows direct normal irradiance, which is the power available on a surface perpendicular to the sun. The deserts in the southwest have the highest irradiance levels while the cloudy northwest has the lowest. The difference is however less than a factor of 2. Image from [49].

rotation around the sun and the earth's own axis into account, the 1365 watts of power blasting the atmosphere becomes:

$$1365 \text{ W} \cdot \frac{1}{2} \cdot \frac{2}{\pi} \cdot 0.96 = 0.96 \cdot \frac{1365}{\pi} \text{ W} \approx 412 \text{ W} \quad (46)$$

Actual power available is also impacted by weather conditions. See figure 21 for a map of solar power measurements in the United States [49]. Figure 21 shows energy in  $\text{kWh}/\text{m}^2/\text{day}$ . Converting to  $\text{watts}/\text{m}^2$  is as simple as dividing by the 24 hours in a day. The southwestern desert's  $7.5 \text{ kWh}/\text{m}^2/\text{day}$  becomes  $312 \text{ W}/\text{m}^2$ , about 75% of the theoretical max. The  $4.0 \text{ kWh}/\text{m}^2/\text{day}$  in the northwest and east becomes  $167 \text{ W}/\text{m}^2$ , about 40% of theoretical max. Notice that the difference between the best and worst locations for solar power in the United States is less than a factor of 2. Since electrical power output scales linearly with the solar power available, the difference in average electrical power generated is also less than 2. More northern latitudes have higher seasonal variations because of the relatively shorter days in winter and longer days in summer, but averaged over the year the difference is less than a factor of 2.

There are two approaches to extracting energy from the sun: thermal and photovoltaic. The thermal approach uses mirrors to direct lots of sunlight to a tube with fluid. Sunlight heats the fluid, heat from the fluid is used to generate steam, and steam turns a turbine to generate electricity. There are many different designs - linear, parabolic, central tower - but they all rely on the same principle. Thermal technology has its niche in generating heat instead of electricity. It can take advantage of a wider spectra of wavelengths from

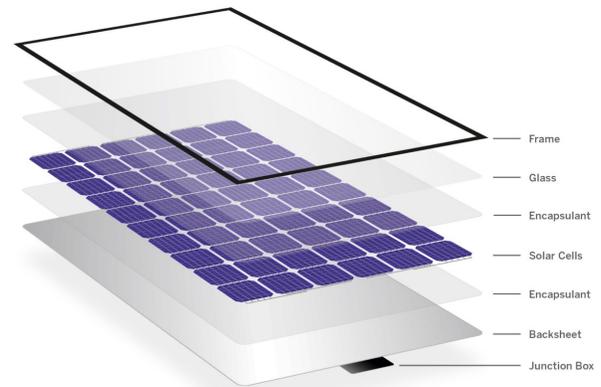


FIG. 22. Components of a solar panel. The solar cells are the blue squares in the middle and the rest of the components are there to protect and connect them. Image from [50].

the sun compared to photovoltaic solar, thus generating heat more efficiently. Converting the heat to electricity decreases the efficiency, increases the cost, and tips the scales in favor of photovoltaic.

Photovoltaic solar is based on semiconductor technology. Silicon is the basis of almost all semiconductor technology. It has 4 valence electrons that all form bonds in pure silicon form. To get some free electrons, phosphorous is added to the mix. Phosphorous has 5 valence electrons. When integrating into the silicon crystal, one valence electron is left unbounded. This is called an n-type semiconductor. To create a solar cell, we also need a p-type semiconductor. P-type is created by doping silicon with Boron. Boron only has 3 valence electrons. When Boron is integrated into the crystal, a hole is left.

When n-type and p-type semiconductors are put next to each other, some of the extra electrons from the n-type semiconductor fall into the holes of the p-type semiconductor. Falling into the holes lowers the energy of the electrons and is thus energetically favorable. However, as electrons enter the holes in the p-type semiconductor, a net negative charge is built up. This charge repels future electrons from falling into holes, putting the system into equilibrium. When light shines on this construction, it can excite an electron into a higher energy state where it's free to move around. The built up charge accelerates the electron in one direction, generating a current. Note that this current always flows in the same direction. Photovoltaic solar is thus one of the few technologies that naturally creates DC current. DC is great for transportation but difficult to construct a grid around. It must thus eventually be converted to AC, adding to the cost of photovoltaic solar.

This combination of n-type and p-type semiconductors

is called a solar cell. A solar panel, as shown in figure 22, contains many solar cells. These cells are what gives the panel its characteristic blue color. Adding many cells together increases the current and voltage output. Other than the cells, solar panels are relatively simple. The rest of the parts in figure 22 are simply there to protect the cells and enable using the power produced by them.

Silicon solar panels today have efficiencies around 20%. One limiting factor on efficiency is that the sun emits a wide spectra of wavelengths. Some wavelengths are not energetic enough to excite electrons, and some are so energetic only part of their energy can be used to excite the electron. This reduces the efficiency of solar panels in converting the energy in the light to electricity. If sunlight only contained one wavelength, we would be able to construct much more efficient panels [51].

Around 95% of solar panels sold today are based on the silicon technology described above. Some other ideas for panels include thin-film photovoltaics, perovskite photovoltaics, organic photovoltaics, quantum dots, multijunction photovoltaics, and concentration photovoltaics, but they are all yet to demonstrate higher cost-efficiency than the current mass-produced silicon panels [52].

## XI. ENERGY TRANSPORTATION

Electrical energy is transported through electric and magnetic fields directed by wires. See figure 23 for a schematic of how a potential difference in a battery can provide energy to a load. Energy goes from a battery to a load in the same way energy moves from the sun to the earth: through the electric and magnetic fields. When most people are introduced to electric circuits, the analogy is made to water flowing in a pipe (or another mechanical system). The flow rate is the current, the inclination is the voltage, and the pipe diameter is the resistance. It is true that electrons drift this way in circuits. However, it's not the net drift of electrons that cause the transfer of energy in the way that the net drift of water does.

Instead, the transfer of energy is caused by the local acceleration from the fields. Connecting a battery to a conducting wire creates a surface charge gradient along the direction of the wire. Since there exists electrons that are free to move all throughout the wire, this surface charge gradient is established with the speed at which the electric field can carry information about connection: the speed of light.

The tendency for like charges to repel equates to the steady-state electric field inside a conductor being 0, causing free negative electrons in the conductor to build up on the surface to cancel the electric field caused by the polarity of the battery, as shown in figure 24. Close to the positive terminal of the battery there will be a net positive charge on the surface of the conductor to cancel the field from the battery, and vice versa at the negative terminal.

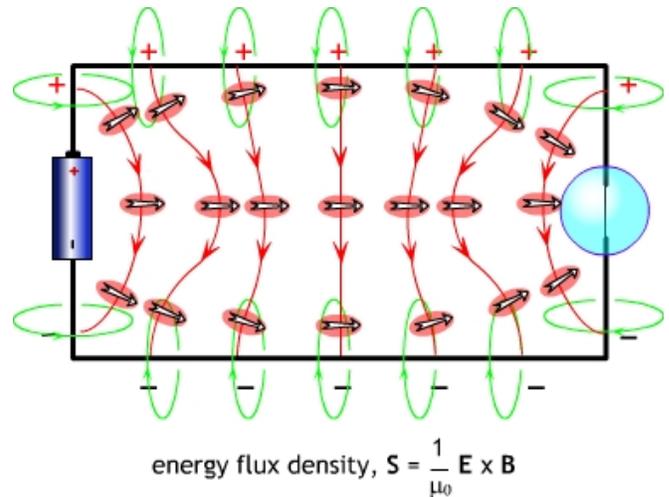


FIG. 23. Power flowing from a battery to a load. The green circles represent the magnetic field created by the moving charges, the red lines are the electric field, and the white arrows are the Poynting vectors. Surface charge are denoted with + and - signs.

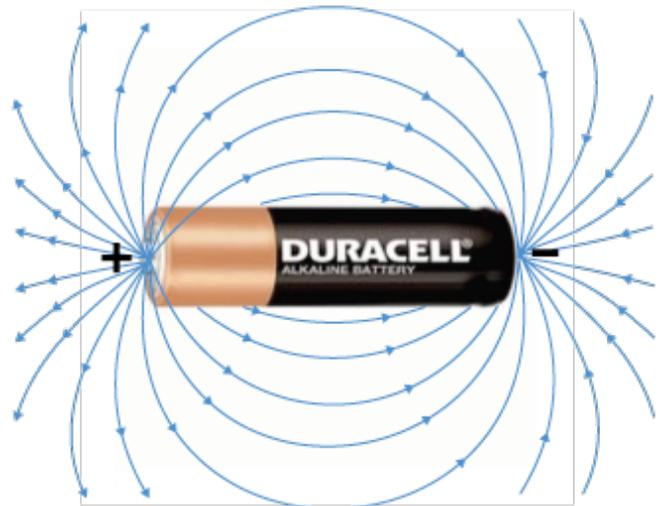


FIG. 24. Electric field from a regular battery. The field arrows correspond to the direction of the force on a positive charge. When placed close to a conductor, the field will cause the charges in the conductor to move around to cancel the field from the battery and create a net zero field inside the conductor.

This gradient of surface charges will, together with the charges on the battery, build up an electric field between the terminals of the battery *inside the conductor*. An electric field inside the conductor is allowed, because the system is *not* at steady-state. Charges on the surface move quickly to cancel the external field because they have a short distance to cover, but cancelling out the charges on the battery poles takes longer time. Steady-

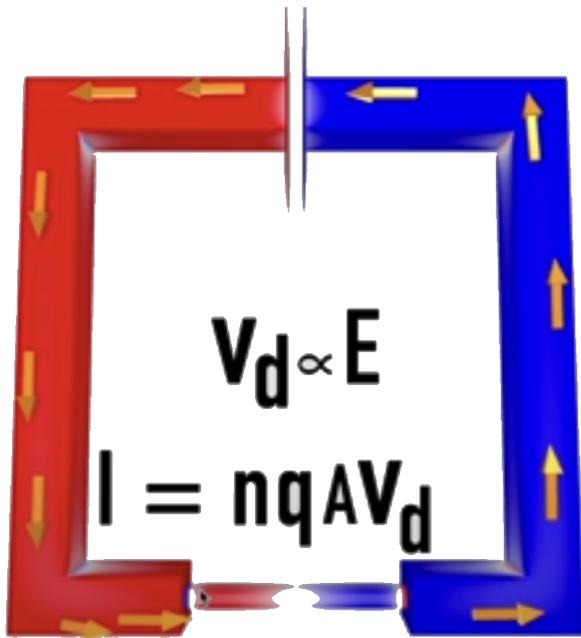


FIG. 25. Illustration of why the voltage drop occurs across a load. A larger gradient of surface charges is required to increase the drift velocity and satisfy the continuity equation.

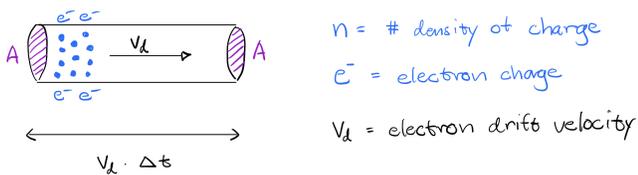


FIG. 26. Electrons drifting through control volume. A net movement of charge through an area constitutes current.

state would be when all the charge is distributed equally in the system, which is clearly not the case as long as the battery is charged.

To discharge the battery, charge has to move from one terminal to the other. The rate of charge movement is called current. Current  $I$  is defined as the charge that flows through an area per unit time. Using the control volume in figure 26, we find the following expression.

$$\vec{I} = \frac{Q}{\Delta t} = \frac{nev_d \Delta t A}{\Delta t} = nev_d A$$

where  $\Delta t$  represents a small time interval,  $n$  the number density of electrons,  $e$  the electron charge, and  $\vec{v}_d$  the drift velocity of the electrons, and  $A$  the cross-sectional area. The net drift velocity of the electrons is in turn proportional to the amount of force pushing them. This force comes from the electric field and can be thought of in aggregate as the voltage difference between the terminals of the circuit. Hence, voltage is proportional

to current:

$$V = IR \quad (47)$$

where  $R$  is a proportionality constant we call resistance. Intuitively, resistance represents how difficult it is for electrons to travel through the wire. Different materials have different resistance. Metals have lower resistance than non-metals and silver, copper, and gold are the lowest among the metals.

Having a low resistance is important to reduce losses when transmitting electricity. However, there is another less intuitive and even more impactful way of reducing transmission losses. The power  $P$  transmitted by a current  $I$  traveling due to a voltage  $V$  can be expressed as:

$$P = IV \quad (48)$$

This can be understood by power being defined as energy per unit time, voltage as energy per unit charge and current as charge per unit time. To find the power dissipated in a wire with resistance  $R$ , we can substitute using 47 and get:

$$P = I^2 R \quad (49)$$

Thus, by decreasing the current, we can also decrease the power dissipated in the wire. We can actually deliver the same power to a consumer using a lower current. Power delivered is the product of the voltage and current. If we double the voltage, we only need half the current. This cuts power losses in the wire by a factor of 4 - a quadratic reduction. Reducing resistance only has a linear impact on the power lost. This is why transmission lines have voltages on the order of 100kV - many orders of magnitude higher than household sockets [4].

Average annual transmission and distribution losses in the United States are around 5% according to the EIA [53]. This might not seem like a lot, but is likely to increase as we transition to renewables. Renewable sites are located further from population centers on average, increasing transportation distances. Further efficiency improvements from voltage increases are unfortunately limited by the ionization of air. At high enough voltages, air becomes conductive and charge leaks through it. These losses start to outweigh the gains from reduced dissipation in cables right around the values of current transmission lines [4].

A way to decrease power losses other than increasing voltage is through using DC current. The alternating electric and magnetic fields of AC current accelerate charges in nearby conductors, resulting in losses. This effect is especially poignant if power lines are under water. Sea water is a relatively good conductor and completely surrounds an underwater cable. DC is thus highly advantageous for underwater transmission. However, there are costs involved in converting between different types of current too. The specifics of the situation and technologies available determine what

is the best choice for each situation, but with transportation distance and DC power generation (from solar power) increasing, we are likely to see an uptick in DC transmission lines in the coming years.

## XII. CONCLUSION

Producing useful forms of energy has been essential for the advancement of our civilization, and it's likely to become even more important in the future. However, new technologies are needed to stop polluting our planet. Understanding the potential and limitations of these technologies is crucial in determining where to invest our limited resources.

While wind and solar provide a clean and cost-competitive way of producing electricity, they suffer from intermittancy. A natural next step would be to investigate and compare the energy storage technologies

available. It would also be interesting to see how stable wind and solar could become when combined and spread across a large area. This will certainly mitigate some of the intermittance problem, but likely not all of it.

Fusion is an incredibly promising technology but it's likely to be too far in the future to fix our current challenges. It's also unclear if it will ever become cost-competitive. An evaluation of the cost in constructing fusion plants and the expected energy returns would show if it makes sense to focus on this technology at all.

Taking the perspective of a developing country would also be an insightful exercise. Much of the expected increase in emissions comes from developing countries. Developing and evaluating technology with this in mind might prove the most useful way of both reducing emissions and improving lives.

Carbon capture technologies might also have an essential role to play in combatting global warming. If we heat up the planet too much before we are able to convert to renewable technologies, it would be necessary to put some of the carbon dioxide back into the ground.

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